XVII. *On the laws of the deviation of magnetized needles towards iron.* By

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THE deviations of a magnetized needle from its natural direction in the plane in which it is constrained to move, due to the action of masses of iron, may be referred to a very simple law, excepting in those cases where the length of the needle bears a very sensible ratio to the distance of the iron. The law is this: if we suppose that the centre of a magnetic particle in the direction of the terrestrial magnetic force, or the centre of a small magnetic needle freely suspended by its centre of gravity, coincides with the centre of the needle whose motion is restricted; that the iron attracts both poles of this particle, or freely suspended needle; and that the whole, or very nearly the whole action takes place on these poles;—then the position of the other needle, in the plane in which it is constrained to move, will be found by referring the freely suspended needle to that plane, by a plane perpendicular to the first. The truth of this being established by experiment, it is very clear that whatever may be the position of a mass of iron, the direction of the deviations of a horizontal or dipping needle due to its action, will be immediately indicated, and a sufficiently simple calculation will give the amount. Several years have elapsed since I first pointed out this law, showing at the same time, by a series of experiments, that the observed deviations are in conformity with it. I have since omitted no opportunity of submitting it to the test of experiment; and taking it as the basis of calculation, have always found, except indeed in cases, as I have before stated, of too great proximity of the disturbing body, that the results which I obtained approximated so closely to the observations, as to leave no doubt in my own mind of its correctness.

However, the truth of this law has latterly been called in question; and, in a paper published in the Transactions of last year, some experiments on horizontal needles, having their magnetism unequally distributed in the two

branches, are described, which are considered by the author as quite decisive of its fallacy *. It is not my intention to enter into an examination of what are there, erroneously I apprehend, considered as the effects that would result from this law, when the equal distribution of magnetism in the two branches of a needle has been disturbed. Immediately after having heard that paper read, I proceeded to ascertain some circumstances which appear to have been overlooked in these experiments, and then noted what, according to the law in question, ought to be the deviations of a needle having either of its branches "deteriorated" when in different positions with respect to an iron shell. Not being then in possession of the experimental results in that paper, I could not compare my conclusions with them; but on doing so when the Part of the Transactions was published, I found them perfectly to accord.

Whatever hypothesis we may adopt for the explanation of the phænomena observed with a needle having its magnetism thus disturbed, or to whatever laws we may apply these phænomena as tests, it is essential that we should know, not only to which end of the needle the disturbing cause has been applied, and its effect on the directive force, but likewise the effect produced on the distribution of the magnetism throughout the needle. It is particularly necessary that the positions of the points where the intensity of the magnetism is the greatest, and of that where the magnetism of contrary names, and on opposite sides balance each other, and which may be termed the magnetic centre of the needle, should be determined. As this had not been done in the experiments to which I have alluded, it became necessary to repeat those experiments, and to determine the positions of these points in the needles employed in that repetition. With this view, I cut three needles of precisely the same form and weight, from the same piece of steel, and applying them over each other, hardened the ends of the three at the same time. These were successively placed in the middle of a groove which two twelve-inch bar magnets exactly fitted, and were each magnetized in precisely the same manner, by passing the ends of the bar magnets, inclined always at the same angle, from centre to ends, the same number of times. One of these was left thus magnetized. The other two were successively placed on a thin board, and the marked

* Phil. Trans. 1827. p. 281.

end of one of the twelve-inch bar magnets was passed twice, on the opposite side of the board, from the centre to the marked end of one needle; and the unmarked end of the magnet, similarly, from the centre to the unmarked end of the other*.

Having determined in these needles the points of greatest intensity, which we may consider as the poles, and likewise the centres of their magnetism, I made observations with them precisely similar to those detailed in the paper already cited†. In the first instance, I proposed to determine the several values of a particular constant, by applying the hypothesis of a central action of the iron to the observations with the needle whose magnetism had been undisturbed, and to employ the mean value in calculating the effects which would arise, according to this hypothesis, from the disturbance of the magnetism in the other needles: but as these values, though not differing greatly from each other, had a pretty regular increase and decrease according to the azimuth of the shell, I considered that the length of the needle might, in this case, bear too great a proportion to the distance of the shell for the proposed law to be strictly applicable, although it might give results approximating to the experiments. The fact however, which is stated to have been ascertained by experiment, that the length of a needle has no sensible influence on the extent of its deviations‡, was opposed to this view; and I therefore compared with each other the values of the constant which would result from these observations, by applying to them a law considered to have been established by experiment: viz. "the tangent of the deviation is proportional to the rectangle of the cosine of the longitude, and the sine of the double latitude" of the needle's centre with reference to that of the shell§.

The values of the constant, thus deduced, differed so widely from each other, that I could have no expectation of obtaining from the mean value, results that should at all approximate to the experiments; and indeed, could entertain no

* These terms, marked end and unmarked end, I employ instead of north end and south end, or south pole and north pole, merely to avoid the ambiguity that might arise in some of the experiments if the name of the end of the magnet or needle had any reference to position.

† Phil. Trans. 1827. p. 284.

‡ Mr. Barlow's Magnetic Attractions, second edition, p. 59.

§ Ibid. p. 39.

doubt of the fallacy of this law, even as approximative. These circumstances led me to suspect the accuracy of the conclusion, that the length of a needle has no sensible influence upon its deviation; and I considered that it would be desirable to ascertain whether such were or were not the fact. This in a theoretical point of view is of some importance; since such a fact would be in direct opposition to the conclusions derived from the theory advocated in the paper in question, and so ably and elaborately developed in M. Poisson's Memoirs on Magnetism. M. Poisson says: "A la vérité, M. Barlow annonce qu'ayant placé successivement dans le même point le milieu de l'aiguille de six pouces, et celui d'une petite aiguille d'un demi-pouce (d'un pouce et demi?) en longueur, il n'a pas observé de différence entre leur déviations; ce qui ferait penser que les deux corrections dont nous parlons, dont l'une a pour effet d'augmenter la déviation, et l'autre, de la diminuer, se seraient à peu près compensées. Mais nous avons lieu de croire que cette compensation a été très-imparfaite; car en calculant les déviations de l'aiguille, sans avoir égard à la double corrections due à sa longueur et à sa force magnétique, les differences que l'on trouve entre le calcul et l'experience, sont trop grandes pour être attribuées en entier aux erreurs des observations*."

These considerations induced me to extend my experiments much beyond the limits which I had originally proposed to myself; and as these experiments were made with great care, and I may fairly state, without any consideration of what might or might not be in conformity with theoretical views, they will afford good tests which may be applied to any theory of magnetism, as well as to the laws with which I have compared them. Previously to detailing any of these experiments, or to entering upon any investigations founded upon the law which they so decidedly establish, I shall briefly notice the facts which I ascertained, and the general explanation they afford, in conformity with that law, of the experiments which are considered by their author to be decisive of its fallacy.

In the first instance, I ascertained that if any bar of steel, uniformly magnetized by the method of double touch, have this state of its magnetism disturbed, by drawing the end of a magnet from its centre to the end of the same

* Mémoire sur la Théorie du Magnetisme, p. 87.

name as that applied to it; that is, drawing the marked end of the magnet towards the marked end of the needle, or the unmarked end towards its unmarked end; then the pole at the end to which the magnet has been applied, or that which has been termed the "deteriorated branch" will approach the centre of the needle. In the other, or "undeteriorated branch," the pole will recede from the centre.

And the magnetic centre will invariably recede from the centre of figure towards the "undeteriorated end," or that to which the disturbing magnet has not been applied. The changes in the positions of these points, in consequence of the disturbance of the magnetism, is best illustrated by the following figures, in which I. II. represent two bar magnets, 8.92 inches in length, 0.16 inch in breadth, 0.09 inch in thickness.

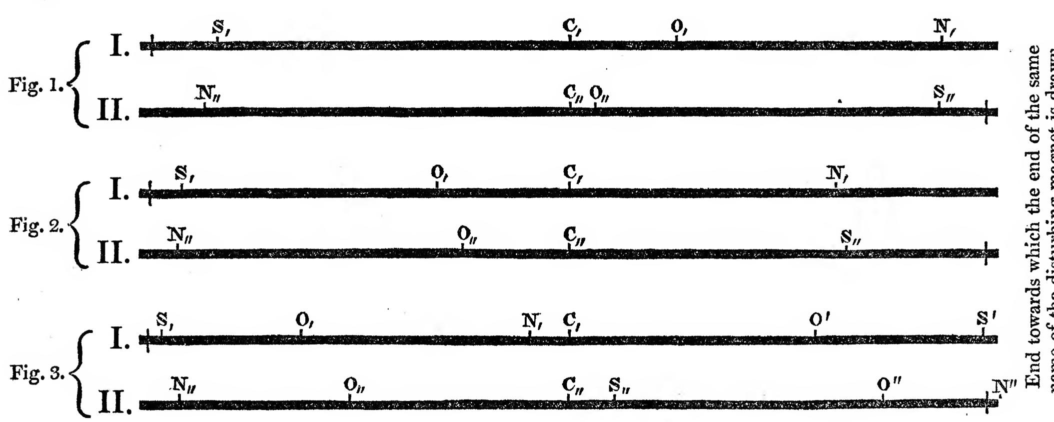

Fig. 1. represents the positions of the poles and magnetic centres after the bars had been magnetized by double touch: $N_{\prime}$, $N_{\prime\prime}$ the north poles; $S_{\prime}$, $S_{\prime\prime}$ the south poles; $C_{\prime}$, $C_{\prime\prime}$ the centres of the bars; $O_{\prime}$, $O_{\prime\prime}$ their magnetic centres.

$C_{\prime}S_{\prime} = 3.74$ inches; $C_{\prime}O_{\prime} = 1.14$ inch; $C_{\prime}N_{\prime} = 3{\cdot}97$ inches;
$C_{\prime\prime}N_{\prime\prime} = 3.84$ inches; $C_{\prime\prime}O_{\prime\prime} = 0.24$ inch; $C_{\prime\prime}S_{\prime\prime} = 3{\cdot}86$ inches.

Fig. 2. represents the positions of these points when the magnetism had been disturbed, by drawing the end of a twelve-inch bar magnet, as before described, once very quickly from the centre to the ends of I. and II.

$C_{\prime}S_{\prime} = 4.09$ inches; $C_{\prime}O_{\prime} = 1.41$ inch; $C_{\prime}N_{\prime} = 2.79$ inches;
$C_{\prime\prime}N_{\prime\prime} = 3.99$ inches; $C_{\prime\prime}O_{\prime\prime} = 1.14$ inch; $C_{\prime\prime}S_{\prime\prime} = 2.91$ inches.

Fig. 3. represents the positions of the same points, and of new poles S′, N″. and magnetic centres O′, O″, in consequence of a further disturbance of the magnetism, by drawing the end of the twelve-inch magnet twice from the centre to the ends of I. and II.

$C_{/}S_{/} = 4.26$ in.; $C_{/}O_{/} = 2.83$ in.; $C_{/}N_{/} = 0.41$ in.; $C_{/}O' = 2.57$ in.; $C_{/}S' = 4.32$ in.;
$C_{//}N_{//} = 4.24$ in.; $C_{//}O_{//} = 2.34$ in.; $C_{//}S_{//} = 0.48$ in.; $C_{//}O'' = 3.31$ in.; $C_{//}N'' = 4.51$ in..

It is easily shown that such changes in the positions of these points will account for what are termed "the secondary deflections," arising from disturbance in the magnetism of the needle, according to the law that the horizontal needle will always assume the position of the projection on the horizontal plane of a needle freely suspended by its centre of gravity, that centre coinciding with the magnetic centre of the horizontal needle, and the iron attracting both poles of the inclined needle.

In the paper to which I have referred it is stated, that when the needle is "placed any where in the magnetic equator of the ball, whichever end of the needle has its magnetism deteriorated, that end will approach the ball."

According to my view of the subject, the centre of the spherical shell will, in this case, coincide with the equator of a freely suspended needle whose centre coincides with the centre of figure, and if the magnetism be equally distributed, of magnetism likewise of the horizontal needle; in which latter case the shell would produce no deviation in the freely suspended needle, and consequently none in the horizontal needle. But when either "branch is deteriorated," I have shown that the magnetic centre of the needle will recede from the centre of figure along the "undeteriorated branch," and the centre of the imaginary suspended needle coinciding with this centre, the centre of the shell will now be in the hemisphere on that side of the equator of the suspended needle in which is the "deteriorated branch." This branch then of the suspended needle, and consequently also, according to the law in question of the horizontal needle, will approach the shell.

The explanation is equally simple in other positions of the needle. The effects are stated to be these: "Generally, in other positions, one branch of the needle will be nearer to the centre of the ball than the other; then, if the near end have its magnetism deteriorated, the needle will approach its natural

meridian; but if the more distant end be deteriorated, the needle will be more deflected, or recede from the meridian."

It is evident that "if the near end have its magnetism deteriorated," the needle's magnetic centre will recede from the shell, and its deviation will nearly correspond to that of another horizontal needle having its magnetism symmetrically distributed in the two branches, and its centre coinciding with the magnetic centre of the former; the deviation will therefore in this case be diminished, or in other words, "the needle will approach its natural meridian." And "if the more distant end be deteriorated," it is clear that the magnetic centre will approach the shell, and consequently "the needle will be more deflected, or recede from the meridian."

This is, I consider, quite sufficient to show the general accordance of these experiments with the law to prove the fallacy of which they were brought forward; and I shall in the subsequent part of this paper, show that this law not only indicates clearly the nature of the deviations which in such cases take place, but that calculations founded upon it, give close approximations to their numerical values.

At the same time that I found the effects which have been described were invariably produced, on the positions of the poles and magnetic centre of the magnetized bars and needles by the disturbance of their magnetism, I likewise found that the intensities of both poles were greatly changed. With the three needles to which I have before referred, and which previous to the disturbance of their magnetism made ten vibrations in about thirty-four seconds, and after it, ten vibrations in about fifty-three seconds; the intensity of the pole in the branch on which the disturbing magnet had been applied, had decreased in about the ratio of five to two, and in the other of seven to four. But these effects I shall more particularly describe when I detail the observations by which I ascertained them.

With regard to the difference in the extent of the deviations of a needle six inches in length, and of one about a third of that length, when successively placed in the same position with respect to the shell, I found that at the distance 16.8 inches from the shell, this difference amounted in some instances to more than $2^\circ\ 30'$; the deviation being in the one case $13^\circ\ 38'$, and in the other $11^\circ\ 04'$: also that there was a sensible, though small difference in the deviations

of the shorter of these needles, and of one only an inch in length; the latter deviation being in the above situation only 10° 51′. I likewise found that even at the distance of twenty-four inches from the shell, the deviations of the longer needle were still sensibly different from those of either of the others. In general, when the needles were near to north or south from the shell's centre, the deviations of the longer needle exceeded those of the shorter; and the reverse took place when the needles were near to east or west from that centre. This is of course a very general description of the effects on the different needles, the precise nature of those effects will be best understood by consulting the tables containing the details of the observations. From this general description, however, we may draw a practical inference of some consequence; viz., that if the deviations that would take place in a needle from the influence of large and distant masses of iron, be counteracted by that of a small mass placed near to the needle, so that it retains its direction in the magnetic meridian in a certain position of the lines joining these masses and the centre of the needle with respect to that meridian; then the needle and these masses preserving their relative positions, if the whole revolve, the deviations caused by the large and distant mass will in some positions preponderate over those due to the smaller, and in some the contrary effect will be produced. It is therefore of importance that the counteracting mass of iron should be at such a distance from the needle that the difference of its effects upon a long needle and upon a very short one would be scarcely appreciable.

Having given a general view of some of the results which I obtained, I shall now deduce, according to the law which I have proposed, equations for the deviations of a horizontal needle due to the action of an iron sphere or shell, applicable to the different circumstances of the experiments which I shall afterwards detail.

Take the centre of the horizontal needle as the origin of three rectangular co-ordinates x, y, z to the centre of the sphere; z being in the direction of the terrestrial force, or the axis of a freely suspended needle concentric with the horizontal needle; y, in that of the magnetic meridian on the plane of the equator of this dipping needle; and x, in that of the intersection of this equator with the horizon: x being measured towards east, y towards north, and z downwards in the direction of the south pole of the dipping needle. Let the

distance of each pole of this dipping needle from its centre be e, or its length $2e$; m the terrestrial magnetic force acting upon each pole in the direction of the dip, and f the force of the sphere upon either pole at the unity of distance, supposing the whole mass collected in the centre. The square of the distance of the sphere's centre from the south pole of the dipping needle will be $x^2 + y^2 + (z - e)^2$; and from its north pole $x^2 + y^2 + (z + e)^2$. So that the forces urging these poles towards the centre of the sphere will be

$$\frac{f}{x^2 + y^2 + (z - e)^2} \text{ and } \frac{f}{x^2 + y^2 + (z + e)^2}.$$

The deviation of the needle will take place in the plane passing through the centre of the sphere and the axis z; and it is evident that it will be the same, if instead of the forces acting upon the north pole, we suppose forces equal to these, but in contrary directions, to act upon the south pole. If then we resolve these forces into others in the direction of the axis z, and perpendicular to it in the plane passing through z and the centre of the sphere, the south pole will be acted upon by a force in the direction z equal to

$$2m + \frac{f.(z - e)}{\{x^2 + y^2 + (z - e)^2\}^{\frac{3}{2}}} - \frac{f.(z + e)}{\{x^2 + y^2 + (z + e)^2\}^{\frac{3}{2}}};$$

and by a force in the direction perpendicular to it equal to

$$f.(x^2 + y^2)^{\frac{1}{2}} \left\{ \frac{1}{\{x^2 + y^2 + (z - e)^2\}^{\frac{3}{2}}} - \frac{1}{\{x^2 + y^2 + (z + e)^2\}^{\frac{3}{2}}} \right\}.$$

Let R be the distance of the sphere's centre from that of the needle, and γ the angle which this distance makes with the axis z. If we substitute R^2 for $x^2 + y^2 + z^2$, $R \cos\gamma$ for z, and $R \sin\gamma$ for $(x^2 + y^2)^{\frac{1}{2}}$, in these expressions, and expand those which result, neglecting the terms containing powers of e higher than the third,

$$2m + \frac{fe}{R^3} \cdot \left[2.(\cos^2\gamma - 1) + \frac{e^2}{R^2} \cdot \{3 - 5\cos^2\gamma(6 - 7\cos^2\gamma)\} \right]$$

will represent the force in the direction z; and

$$\frac{fe}{R^3} \cdot \sin\gamma\cos\gamma \left\{ 2.3 - \frac{e^2}{R^2}(3.5 - 5.7\cos^2\gamma) \right\},$$

the force perpendicular to z, in the plane passing through the axis z and the centre of the sphere.

If then ψ is the angle of deviation of the needle from the axis z, ψ will also be the angle which the resultant of these forces makes with that axis: we shall therefore have

$$\operatorname{Tan} \psi = \frac{\sin \gamma \cos \gamma \left\{ 2.3 - \frac{e^2}{R^2} (3.5 - 5.7 \cos^2 \gamma) \right\}}{\frac{2 m R^3}{f e} + 2 (3 \cos^2 \gamma - 1) + \frac{e^2}{R^2} \{ 3 - 5 \cos^2 \gamma (6 - 7 \cos^2 \gamma) \}} \qquad (1)$$

If e is extremely small, we may neglect the terms containing $\frac{e^2}{R^2}$, and the equation then becomes

$$\operatorname{Tan} \psi = \frac{3 \sin \gamma \cos \gamma}{\frac{m R^3}{f e} + 3 \cos^2 \gamma - 1} \qquad (2)$$

or

$$\operatorname{Tan} \psi = \frac{3 \sin 2 \gamma}{\frac{2 m R^3}{f e} + 3 \cos 2 \gamma + 1} \qquad (3)$$

These equations give the position which an extremely short freely suspended needle would assume by the combined action of the iron sphere, and of terrestrial magnetism; and according to the law which I consider obtains, the projection of this inclined needle upon the horizontal plane will give the position which the horizontal needle will assume in consequence of the same action*.

The rectangular co-ordinates x, y, z determine the position of the iron sphere; but if ω be the angle which the projection of the radius vector R upon the plane $x\,y$ makes with the axis y, its position will be determined by the polar co-ordinates R, γ, ω. Let the angle which the axis z makes with the vertical, or the complement of the dip, be represented by η, and the angle which the projection of the freely suspended needle upon the horizontal plane makes

* This principle, that the deviations of the horizontal needle may be referred to those of the inclined needle, I had pointed out a considerable time previous to my showing, in a paper in the Camb. Phil. Trans. for 1820, that it is consistent with experimental results. It was adopted in the first edition of Mr. BARLOW's "Essay on Magnetic Attactions," and likewise in the theoretical investigations in the second edition of that work. M. POISSON, in his "Mémoire sur la Théorie du Magnetisme," published in 1824, employs the same principle.

with the projection of the axis z upon the same plane or the magnetic meridian, and which, according to the principle adopted, is the deviation of the horizontal needle, be represented by ϕ: then the intersection of the axis z, of the vertical, and of the line of the inclined needle with the surface of a sphere described about the centre of the needle, will be the angular points of a spherical triangle, of which two sides are η, ψ, their included angle $\pi - \omega$, and ϕ the angle opposite to ψ. We shall therefore have

$$\sin \omega \, . \cot \phi = \sin \eta \cot \psi + \cos \omega \cos \eta ;$$

from which and the equation (3), we obtain

$$\text{Cot } \phi = \frac{\sin \eta \left\{ \frac{2 m \text{R}^3}{fe} + 3 \cos 2 \gamma + 1 \right\}}{3 \sin 2 \gamma \sin \omega} + \cos \eta \cot \omega. \qquad (4)$$

If we determine the position of the iron sphere by polar co-ordinates relative to the vertical and the plane of the horizon, calling the angle which the radius R makes with the vertical v, and the angle which its projection on the plane of the horizon makes with the magnetic meridian, or the azimuth of the iron sphere, θ; then v, γ, η will be the sides of a spherical triangle of which the angles opposite to v and γ are $\pi - \omega$ and θ. We have therefore

$$\cos \gamma = \cos \theta \, . \sin \eta \, . \sin v + \cos \eta \, . \cos v,$$
$$\sin \omega \, . \sin \gamma = \sin \theta \, . \sin v,$$
$$\cot \theta \sin \omega = \cot \gamma \sin \eta + \cos \omega \cos \eta.$$

From these, putting the equation (4) in the form

$$\text{Cot } \phi = \frac{\sin \eta \, . \left(\frac{m \text{R}^3}{fe} - 1 \right)}{3 \sin \gamma \cos \gamma \sin \omega} + \frac{\cot \gamma \sin \eta + \cos \omega \cos \eta}{\sin \omega},$$

we obtain

$$\text{Cot } \phi = \frac{\frac{m \text{R}^3}{fe} - 1}{3 \sin \theta \sin v (\cos \theta \sin v + \cot \eta \cos v)} + \cot \theta. \qquad (5)$$

If, as is most convenient for experiment, the position of the iron sphere be determined by its vertical distance below the horizontal plane on which the needle is, and its horizontal distance from the centre of the needle, let r be the projection of R on the horizontal plane, and h the distance of the sphere's centre below that plane: then since, $\sin v = \frac{r}{\text{R}}$, and $\cos v = \frac{h}{\text{R}}$,

$$\text{Cot}\,\varphi = \frac{\left(\frac{m\,\text{R}^3}{fe} - 1\right).\,\text{R}^2}{3\,r\sin\theta\,(r\cos\theta + h\cot\eta)} + \cot\theta \qquad (6)$$

This when $h = 0$, or the centre of the sphere is in the same horizontal plane as that of the needle, becomes

$$\text{Cot}\,\varphi = \frac{2\left(\frac{m\,\text{R}^3}{fe} - 1\right)}{3\sin 2\,\theta} + \cot\theta \qquad (7)$$

from which the values of $\frac{m}{fe}$ will be determined in different magnetic latitudes from the observed values of φ, independent of the dip.

We may, in a similar manner, deduce an equation for the deviations of a needle in which the magnetism has been disturbed by applying to one of its poles the corresponding pole of a magnet; and thence determine the deviations of a needle having its magnetism symmetrically distributed in its two branches from the observed deviations of a needle in which such distribution has been disturbed. I have already stated that under these circumstances the intensities of both poles were changed, and it will be seen by the experiments detailed in a subsequent part of this paper, that their distances from the new magnetic centre were not the same, the more energetic pole being the nearer; and I will assume that the same is the case with the needle freely suspended from a point which, as before, is its centre of gravity and of magnetism. Let the quantities which before were indicated by m, f, e be indicated by m', f', e' for the north or upper pole of the suspended needle, and by $m_{\prime}, f_{\prime}, e_{\prime}$ for its south or lower pole. We will also suppose that the south pole, or that which is usually marked, is that to which the corresponding pole of a magnet has been applied, or that "the deteriorated branch" is the northern; and that $\text{R}_{\prime}$ is the distance of the sphere's centre from the magnetic centre of this needle, $\gamma_{\prime}$ the angle which $\text{R}_{\prime}$ makes with the axis z passing through this centre, and $\psi_{\prime}$ the angle which the freely suspended needle makes with the same axis, or its deviation due to the action of the iron sphere. The equation corresponding to (2) will now become

$$\text{Tan}\,\psi_{\prime} = \frac{\sin\gamma_{\prime}\,\{\text{R}_{\prime}(f'e' - f_{\prime}e_{\prime}) + 3\cos\gamma_{\prime}\,(f'e'^2 + f_{\prime}e_{\prime}^2)\}}{(m'e' + m_{\prime}e_{\prime}).\,\text{R}_{\prime}^3 + \cos\gamma_{\prime}\,\{\text{R}_{\prime}(f'e' - f_{\prime}e_{\prime}) + 3\cos\gamma_{\prime}(f'e'^2 + f_{\prime}e_{\prime}^2)\} - (f'e'^2 + f_{\prime}e_{\prime}^2)}$$

Now $\frac{m'}{f'} = \frac{m_{\prime}}{f_{\prime}} = \frac{m}{f}$; and it will be seen by the experiments that the values

of the products of the intensity of each pole by its distance from the magnetic centre are so nearly the same, that, as an approximation, we may assume them equal; and supposing the same distribution in the inclined neeedle, we have $\frac{e'}{e_{\prime}} = \frac{f_{\prime}}{f'} = \frac{m_{\prime}}{m'}$. Consequently, $\operatorname{Tan} \psi_{\prime} = \frac{3 \sin \gamma_{\prime} \cos \gamma_{\prime}}{\frac{2e}{e_{\prime} + e'} \cdot \frac{m R_{\prime}^3}{fe} + 3 \cos^2 \gamma_{\prime} - 1}$ (8)

If then $\theta_{\prime}$ is the azimuth of the iron sphere's centre from the magnetic centre of the deteriorated horizontal needle; $\varphi_{\prime}$ the deviation of this needle from the magnetic meridian; and $r_{\prime}$ the projection of $R_{\prime}$ upon the horizontal plane; we have, putting p for $\frac{2e}{e_{\prime} + e'}$,

$$\operatorname{Cot} \varphi_{\prime} = \frac{\left(\frac{p m R_{\prime}^3}{fe} - 1\right) \cdot R_{\prime}^2}{3 r_{\prime} \sin \theta_{\prime} (r_{\prime} \cos \theta_{\prime} + h \cot \eta)} + \cot \theta_{\prime} \qquad (9)$$

Let k be the distance of the magnetic centre of the deteriorated needle from its centre of figure, which distance, as will be seen from the experiments, is to be measured towards the south when the northern branch of the needle is that "deteriorated;" then

$$r_{\prime}^2 = r^2 + k^2 + 2 r k \cos(\theta - \varphi_{\prime}); \quad R_{\prime}^2 = R^2 + k^2 + 2 r k \cos(\theta - \varphi_{\prime});$$
$$r_{\prime} \sin \theta_{\prime} = r \sin \theta + k \sin \varphi_{\prime}; \quad r_{\prime} \cos \theta_{\prime} = r \cos \theta + k \cos \varphi_{\prime}.$$

Substituting these values in the equation (9), and limiting the approximation to the first power of $\frac{k}{r}$, we have,

$$\operatorname{Cot} \varphi_{\prime} = \frac{p m R^3}{fe} \cdot \frac{R^2}{3 r Q} \cdot \left\{ 1 + \left(\frac{5 r^2}{R^2} \cdot \cos(\theta - \varphi_{\prime}) - \frac{S}{Q} \right) \cdot \frac{k}{r} \right\} - \frac{R^2}{3 r Q} \left\{ 1 + \left(\frac{2 r^2}{R^2} \cdot \cos(\theta - \varphi_{\prime}) - \frac{S}{Q} \right) \cdot \frac{k}{r} \right\} + \cot \theta + \frac{\sin(\theta - \varphi_{\prime})}{\sin^2 \theta} \cdot \frac{k}{r}$$

where $Q = \sin \theta \cdot (r \cos \theta + h \cot \eta)$, and $S = r \sin(\theta + \varphi_{\prime}) + h \cot \eta \sin \varphi_{\prime}$.

Putting for $\frac{p m R^3}{fe} \cdot \frac{R^2}{3 r Q}$ its value from the equation (6), $\cot \varphi - \cot \theta + \frac{R^2}{3 r Q}$, we obtain

$$\operatorname{Cot} \varphi = \frac{\cot \varphi_{\prime} + (p-1) \cdot \left\{ \cot \theta - \frac{R^2}{3 r Q} \right\} + \left[p \cdot \left\{ \cot \theta - \frac{R^2}{3 r Q} \right\} \cdot \left\{ \frac{5 r^2}{R^2} \cos(\theta - \varphi_{\prime}) - \frac{S}{Q} \right\} + \frac{R^2}{3 r Q} \cdot \left\{ \frac{2 r^2}{R^2} \cdot \cos(\theta - \varphi_{\prime}) - \frac{S}{Q} \right\} - \frac{\sin(\theta - \varphi_{\prime})}{\sin^2 \theta} \right] \cdot \frac{k}{r}}{p \cdot \left[1 + \left\{ \frac{5 r^2}{R^2} \cos(\theta - \varphi_{\prime}) - \frac{S}{Q} \right\} \cdot \frac{k}{r} \right]},$$

or

$$\operatorname{Cot} \varphi = \frac{\cot \varphi_{\prime} - \left(\cot \theta - \frac{R^2}{3 r Q} \right) + \left[\frac{R^2}{3 r Q} \cdot \left\{ \frac{2 r^2}{R^2} \cdot \cos(\theta - \varphi_{\prime}) - \frac{S}{Q} \right\} - \frac{\sin(\theta - \varphi_{\prime})}{\sin^2 \theta} \right] \cdot \frac{k}{r}}{p \cdot \left[1 + \left\{ \frac{5 r^2}{R^2} \cdot \cos(\theta - \varphi_{\prime}) - \frac{S}{Q} \right\} \cdot \frac{k}{r} \right]} + \cot \theta - \frac{R^2}{3 r Q} \qquad (10),$$

the latter being rather more convenient for computation.

From this equation the values of φ, or the deviations of a needle having its

magnetism similarly distributed in its two branches, may be computed from the observed values of $\varphi_{\prime}$, or the observed deviations of a needle having its magnetism dissimilarly distributed.

The equation (10) admits of considerable simplification when the centre of the shell is in the same horizontal plane as that of the needle. In this case $h = 0$; r becomes R; $Q = R \sin\theta \cos\theta$; $S = R \sin(\theta + \varphi_{\prime})$; and consequently,

$$\operatorname{Cot}\varphi = \frac{\cot\varphi_{\prime} - \frac{3\cos^2\theta - 1}{3\sin\theta\cos\theta} + \left[\frac{1}{3\sin\theta\cos\theta}\left\{2\cos(\theta - \varphi_{\prime}) - \frac{\sin(\theta + \varphi_{\prime})}{\sin\theta \,.\, \cos\theta}\right\} - \frac{\sin(\theta - \varphi_{\prime})}{\sin^2\theta}\right] . \frac{k}{R}}{p . \left[1 + \left\{5\cos(\theta - \varphi_{\prime}) - \frac{\sin(\theta + \varphi_{\prime})}{\sin\theta\cos\theta}\right\} . \frac{k}{R}\right]} + \frac{3\cos^2\theta - 1}{3\sin\theta\cos\theta},$$

or since $$2\cos(\theta - \varphi_{\prime}) - \frac{\sin(\theta + \varphi_{\prime})}{\sin\theta\cos\theta} = 2\cot 2\theta \,.\, \sin(\theta - \varphi_{\prime}),$$

$$\operatorname{Cot}\varphi = \frac{\cot\varphi_{\prime} - \frac{3\cos 2\theta + 1}{3\sin 2\theta} - \frac{2(\cos 2\theta + 3)}{3\sin^2 2\theta} . \sin(\theta - \varphi_{\prime}) . \frac{k}{R}}{p . \left[1 + \left\{3\cos(\theta - \varphi_{\prime}) + 2\cot 2\theta \,.\, \sin(\theta - \varphi_{\prime})\right\} . \frac{k}{R}\right]} + \frac{3\cos 2\theta + 1}{3\sin 2\theta} \qquad (11)$$

If φ and $\varphi_{\prime}$ have both been determined from observation, the value of p may be determined from this equation,

$$p = \frac{\cot\varphi_{\prime} - \frac{3\cos 2\theta + 1}{3\sin 2\theta} - \frac{2(\cos 2\theta + 3)}{3\sin^2 2\theta} . \sin(\theta - \varphi_{\prime}) . \frac{k}{R}}{\left\{\cot\varphi - \frac{3\cos 2\theta + 1}{3\sin 2\theta}\right\} . \left[1 + \left\{3\cos(\theta - \varphi_{\prime}) + 2\cot 2\theta \,.\, \sin(\theta - \varphi_{\prime})\right\} . \frac{k}{R}\right]} \qquad (12)$$

The equations (10), (11), (12) have been obtained on the supposition that the disturbing magnet has been applied to the northern branch of the needle, or that the northern branch is "deteriorated," in which case the magnetic centre will be found between the centre of figure and the southern extremity; but if the disturbing magnet is applied to the southern branch, or it is this branch which is "deteriorated," the magnetic centre will be found between the centre of figure and the northern extremity; and therefore in applying these equations to the latter case, k must be considered negative.

Having deduced equations by which the deviations of a horizontal needle, due to the action of an iron sphere or shell, may be computed, both when the distribution of magnetism is similar, and when it is dissimilar in the two branches of the needle, I shall now proceed to the detail of experiments, and to their comparison with these theoretical results.

I have already stated that circumstances led me to suspect the accuracy of

the observations from which the conclusion had been drawn, that a mass of soft iron will cause the same deviation in a magnetized needle of six inches, and in one of an inch and a half in length; or in other words, that, within these limits, the deviations are independent of the needle's length; and the first observations which I made, not only confirmed my suspicions, but placed the matter beyond doubt. The apparatus which I made use of is nearly the same as that which I employed for the experiments by which I first showed, that the deviations of a horizontal needle due to the action of a mass of iron are in conformity with the law in question. In those experiments I employed a shell 12.78 inches in diameter: the diameter of that used in the experiments I am about to describe is 17.7 inches. I was indebted for the use of this shell to Sir William Congreve, who allowed me to select it from a few of that diameter in the Repository at Woolwich. This shell can be lowered or raised by means of pulleys, and passes through a circular hole in a horizontal table. On this table I had described two circles concentric with the shell, when its centre was in the plane of the table, one 9.5 inches radius, the other 28 inches; and these circles I had divided very carefully to thirds of a degree: so that, a very fine wire being stretched from a division in the one to the corresponding division in the other, the compass can be adjusted with considerable accuracy to any azimuth from the shell's centre, by means of indexes outside the box, corresponding to 0 and 180 on the divided ring. This ring is divided to 20′; and as I had made the needles to fit it very accurately, and had terminated them by very sharp points, I consider that I could read the deviations pretty accurately to 2′ by means of a lens; and as in all cases, the readings at both ends of the needle were registered and a mean taken, the errors of centering were avoided, and the whole amount of error diminished. When the shell had been lowered so that its vertical distance above or below the plane of the table was that at which the observations were to be made, I very carefully adjusted it, by means of plummets suspended from very fine wires crossing over it, so that the vertical through its centre passed through the centre of the concentric graduated circles on the horizontal table.

To determine the influence which the length of a needle has on its deviations, I made use of three different needles, which I had marked A, D, P. A is 6.01 inches in length, bounded by circular arcs, its greatest breadth being 0.52 inch; D is 1.87 inch in length, its breadth 0.22 inch; P is 1.03 inch in

length, and 0.19 inch in breadth; the forms of D and P being nearly similar to that of A. The weight of A is 77.5 grains; of D 7.0 grains; and of P 7.75 grains. To the needles D and P were attached slips of mica, of the same length as A, having fine lines drawn on them, and with which the axes of the needles were made to coincide; so that I could observe the corresponding deviations of the three needles by placing them successively on the pivot of the same compass-box, which therefore remained in the same position for each, after it had been so adjusted that the centre of the shell had the required azimuth with respect to that of the needle. With these needles, observations were made in three different positions of the shell; viz. when its centre was ten inches above the horizontal plane passing through the needle's centre; when its centre coincided with; and when ten inches below that plane. When the shell was above or below that plane, the horizontal distance of the needle's centre from the vertical passing through that of the shell was 13.5 inches; so that placing the needle in different azimuths on the horizontal table, had the same effect as making the shell describe a parallel to the horizon in a sphere 16.8 inches radius described about the needle. When the centre of the shell was in the horizontal plane passing through the needle's centre, the distance between these centres was 16.8 inches. In all these cases the deviations of the three needles were observed at every 20° of azimuth round the circle. These distances were adopted as they are those selected in the experiments described in the paper, "On the secondary deflections produced in a magnetized needle by an iron shell, &c." in the Transactions of last year; and I had consequently computed the values of the angles γ, ω, in the equations (3), (4), according to such adjustments. Whatever errors might be made in the adjustment of the compass,—and I have no hesitation in saying that these were always of small amount,—the relative deviations of the three needles for each azimuth were independent of these errors, since the compass-box remained in the same position for the corresponding observations with each needle.

The following Tables contain the deviations of the north end of the needle, deduced from the readings at both ends, in three different sets of experiments; in each of which all the adjustments of the height of the shell, its centering, the azimuth and distance of the compass, were made afresh. The mean of the deviations corresponding to each azimuth of the shell to the west is taken, and likewise of those corresponding to each azimuth to the east;

and again the mean of these mean deviations in opposite directions for each azimuth. In these, + indicates that the deviation of the north end of the needle is of the same name as the azimuth of the shell, or that it is towards the shell; and −, that it is of a contrary name, or from the shell.

The centre of the Shell 10 inches below the horizontal plane passing through the Needle's centre, and its distance from that centre 16.8 inches.

Needle of which the deviation was observed.	Azimuth of the Shell's centre from North.	The Azimuth of the Shell's centre from that of the Needle being West.				East.				Means of the deviations with the Shell West and with the Shell East.
		Deviations of the North end.			Means.	Deviations of the North end.			Means.	
A. Length = 6.01 inches.	20°	10° 40′W	10° 46′W	10° 48′W	10° 44′ 40″W	10° 40′E	10° 32′E	10° 33′E	10° 35′ 00″E	+10° 39′ 50″
	40	20 24	20 34	20 30	20 29 20	20 24	20 14	20 17	20 18 20	20 23 50
	60	28 02	28 06	28 10	28 06 00	28 05	27 55	27 55	27 58 20	28 02 10
	80	32 23	32 27	32 29	32 26 20	32 47	32 41	32 46	32 44 40	32 35 30
	100	32 00	32 08	32 08	32 05 20	32 40	32 35	32 27	32 34 00	32 19 40
	120	27 50	28 09	27 58	27 59 00	28 18	28 18	28 08	28 14 40	28 06 50
	140	22 02	22 11	21 56	22 03 00	22 19	22 21	22 11	22 17 00	22 10 00
	160	13 34	13 46	13 21	13 33 40	13 32	13 31	13 18	13 27 00	13 30 20
D. Length = 1.87 inch.	20	9 50	10 00	9 57	9 55 40	9 54	9 47	9 46	9 49 00	9 52 20
	40	19 01	19 10	19 02	19 04 20	19 05	18 56	18 52	18 57 40	19 01 00
	60	26 32	26 36	26 34	26 34 00	26 31	26 27	26 22	26 26 40	26 30 20
	80	31 31	31 36	31 35	31 34 00	31 49	31 46	31 45	31 46 40	31 40 20
	100	32 51	32 57	32 52	32 53 20	33 16	33 12	33 06	33 11 20	33 02 20
	120	29 29	29 44	29 31	29 34 40	29 42	29 44	29 35	29 40 20	29 37 30
	140	21 36	21 49	21 36	21 40 20	21 51	21 51	21 53	21 51 40	21 46 00
	160	10 58	11 10	10 59	11 02 20	11 07	10 58	10 56	11 00 20	11 01 20
P. Length = 1.03 inch.	20	9 51	9 56	9 54	9 53 40	9 50	9 42	9 48	9 46 40	9 50 10
	40	18 57	19 07	18 57	19 00 20	19 02	18 49	18 53	18 54 40	18 57 30
	60	26 28	26 34	26 28	26 30 00	26 30	26 20	26 24	26 24 40	26 27 20
	80	31 31	31 40	31 30	31 33 40	31 53	31 41	31 48	31 47 20	31 40 30
	100	32 57	33 03	32 56	32 58 40	33 23	33 21	33 14	33 19 20	33 09 00
	120	29 36	29 54	29 33	29 41 00	29 46	29 49	29 46	29 47 00	29 44 00
	140	21 46	21 49	21 32	21 42 20	21 46	21 49	21 56	21 50 20	21 46 20
	160	10 55	11 00	10 48	10 54 20	10 46	10 42	10 40	10 42 40	10 48 30

The centre of the Shell in the horizontal plane passing through the Needle's centre, and its distance from that centre 16.8 inches.										
Needle of which the deviation was observed.	Azimuth of the Shell's centre from North.	The Azimuth of the Shell's centre from that of the Needle being								Means of the deviations with the Shell West and with the Shell East.
		West.				East.				
		Deviations of the North end.			Means.	Deviations of the North end.			Means.	
A. Length = 6.01 inches.	20°	7° 14′ W	7° 16′ W	7° 20′ W	7° 16′ 40″ W	7° 07′ E	7° 16′ E	7° 13′ E	7° 12′ 00″ E	+ 7° 14′ 20″
	40	11 45	11 46	11 52	11 47 40	11 41	11 34	11 35	11 36 40	11 42 10
	60	10 20	10 33	10 24	10 25 40	10 06	10 03	10 06	10 05 00	10 15 20
	80	3 46	3 55	3 53	3 51 20	3 30	3 30	3 22	3 27 20	3 39 20
	100	4 30E	4 21E	4 24E	4 25 00E	4 42W	4 34W	4 39W	4 38 20W	− 4 31 40
	120	11 05	11 04	10 58	11 02 20	11 22	11 16	11 28	11 22 00	11 12 10
	140	12 07	12 07	12 04	12 06 00	12 13	12 17	12 10	12 13 20	12 09 40
	160	7 20	7 19	7 20	7 19 40	7 38	7 45	7 38	7 40 20	7 30 00
D. Length = 1.87 inch.	20	6 13W	6 17W	6 22W	6 17 20W	6 07E	6 11E	6 08E	6 08 40E	+ 6 13 00
	40	10 26	10 34	10 39	10 33 00	10 29	10 18	10 18	10 21 40	10 27 20
	60	10 17	10 26	10 24	10 22 20	9 59	9 58	9 58	9 58 20	10 10 20
	80	4 24	4 40	4 38	4 34 00	4 01	4 09	4 02	4 04 00	4 19 00
	100	4 40E	4 27E	4 19E	4 28 40E	4 54W	4 44W	4 53W	4 50 20W	− 4 39 30
	120	10 27	10 22	10 19	10 22 40	10 41	10 40	10 43	10 41 20	10 32 00
	140	10 25	10 24	10 23	10 24 00	10 34	10 38	10 38	10 36 40	10 30 20
	160	6 06	6 04	6 05	6 05 00	6 31	6 39	6 36	6 35 20	6 20 10
P. Length = 1.03 inch.	20	6 12W	6 12W	6 16W	6 13 20W	6 01E	6 12E	6 13E	6 08 40E	+ 6 11 00
	40	10 24	10 28	10 34	10 28 40	10 20	10 18	10 22	10 20 00	10 24 20
	60	10 10	10 25	10 17	10 17 20	9 55	10 04	10 05	10 01 20	10 09 20
	80	4 29	4 37	4 36	4 34 00	4 04	4 14	4 05	4 07 40	4 20 50
	100	4 37E	4 28E	4 26E	4 30 20E	5 00W	4 47W	4 50W	4 52 20W	− 4 41 20
	120	10 21	10 21	10 18	10 20 00	10 41	10 33	10 37	10 37 00	10 28 30
	140	10 16	10 21	10 25	10 20 40	10 30	10 29	10 26	10 28 20	10 24 30
	160	5 59	6 03	6 06	6 02 40	6 21	6 29	6 25	6 25 00	6 13 50

The centre of the Shell 10 inches above the horizontal plane passing through the Needle's centre, and its distance from that centre 16.8 inches.

Needle of which the deviation was observed.	Azimuth of the Shell's centre from North.	The Azimuth of the Shell's centre from that of the Needle being West. Deviations of the North end.			West. Means.	East. Deviations of the North end.			East. Means.	Means of the deviations with the Shell West and with the Shell East.
A. Length = 6.01 inches.	20°	13° 00′E	12° 51′E	13° 04′E	12° 58′ 20″E	12° 32′W	12° 13′W	12° 32′W	12° 25′ 40″W	−12° 42′ 00″
	40	21 41	21 26	21 34	21 33 40	21 07	20 40	21 10	20 59 00	21 16 20
	60	28 09	28 03	27 57	28 03 00	27 32	27 04	27 35	27 23 40	27 43 20
	80	32 38	32 34	32 15	32 29 00	31 51	31 27	31 50	31 42 40	32 05 50
	100	32 54	32 54	32 43	32 50 20	32 12	31 56	32 13	32 07 00	32 28 40
	120	28 04	28 16	28 06	28 08 40	27 54	27 50	27 56	27 53 20	28 01 00
	140	20 06	20 04	20 02	20 04 00	20 04	20 07	20 09	20 06 40	20 05 20
	160	10 16	10 08	10 13	10 12 20	10 43	10 44	10 43	10 43 20	10 27 50
D. Length = 1.87 inch.	20	11 00	10 52	10 59	10 57 00	10 39	10 20	10 25	10 28 00	10 42 30
	40	21 31	21 19	21 20	21 23 20	21 04	20 34	20 55	20 51 00	21 07 10
	60	29 49	29 46	29 21	29 38 40	29 13	28 44	29 19	29 05 20	29 22 00
	80	33 16	33 18	32 54	33 09 20	32 38	32 07	32 43	32 29 20	32 49 20
	100	31 50	31 50	31 40	31 46 40	31 27	31 04	31 25	31 18 40	31 32 40
	120	26 30	26 35	26 28	26 31 00	26 20	26 20	26 32	26 24 00	26 27 30
	140	18 39	18 38	18 35	18 37 20	18 38	18 47	18 50	18 45 00	18 41 10
	160	9 36	9 21	9 19	9 25 20	9 50	10 00	9 59	9 56 20	9 40 50
P. Length = 1.03 inch.	20	11 05	10 59	11 02	11 02 00	10 24	10 11	10 12	10 15 40	10 38 50
	40	21 31	21 26	21 20	21 25 40	21 01	20 35	20 53	20 49 40	21 07 40
	60	29 41	29 50	29 31	29 40 40	29 17	28 36	29 10	29 01 00	29 20 50
	80	33 21	33 15	33 03	33 13 00	32 31	32 08	32 33	32 24 00	32 48 30
	100	31 40	31 39	31 36	31 38 20	31 15	31 01	31 18	31 11 20	31 24 50
	120	26 23	26 26	26 23	26 24 00	26 09	26 13	26 14	26 12 00	26 18 00
	140	18 31	18 30	18 34	18 31 40	18 38	18 43	18 41	18 40 40	18 36 10
	160	9 24	9 19	9 22	9 21 40	9 56	9 55	9 54	9 55 00	9 38 20

I did not consider it necessary to make corresponding observations to these at an increased distance between the centres of the shell and needle at every 20° of azimuth; I however made the following observations in those positions where the differences in the deviations of the needles are the greatest,

when that distance was increased to twenty-four inches. The results show that at this distance the differences, though small, are still appreciable.

Needle of which the deviation was observed.	Azimuth of the Shell's centre from North.	The centre of the Shell 24 inches from that of the Needle, and 14.29 inches below the horizontal plane passing through the centre of the Needle.			14.29 inches above the horizontal plane passing through the centre of the Needle.			In the horizontal plane passing through the centre of the Needle.			
		Azimuth West. Deviation.	Azimuth East. Deviation.	Mean Deviation.	Azimuth West. Deviation.	Azimuth East. Deviation.	Mean Deviation.	Azimuth of Shell's centre from North.	Azimuth West. Deviation.	Azimuth East. Deviation.	Mean Deviation.
A.	20°	10° 45′W	10° 50′E	+10° 47½′	2° 47′E	2° 39′W	− 2° 43′				
	60	8 10	8 13	8 11½	8 22	8 04	8 13	40°	3° 48′W	3° 55′E	+3° 51½′
	120	2 55	2 42	2 48½	11 07	11 02	11 04½	140	3 57E	4 00W	−3 58½
D.	20	10 29	10 52	10 40½	2 34	2 22	2 28				
	60	8 10	8 18	8 14	8 29	7 57	8 13	40	3 33W	3 57E	+3 45
	120	2 40	2 25	2 32½	10 59	10 46	10 52½	140	3 44E	3 40W	−3 42
P.	20	10 10	11 09	10 39½	2 38	2 15	2 26½				
	60	7 52	8 35	8 13½	8 49	7 41	8 15	40	3 09W	4 18E	+3 43½
	120	2 38	2 27	2 32½	11 18	10 27	10 52½	140	3 04E	3 21W	−3 42½

The differences in the deviations of the three needles in the foregoing tables may perhaps partly be attributed to the difference of their masses; and this probably produced some slight effect, especially at the nearer distance. This effect, arising from the shell becoming magnetic by the influence of the needle itself, would be in all cases to make the nearer end of the needle deviate towards the shell; and consequently in the position where the difference in the deviations of the long and short needles was observed to be greatest, it would be contrary to that observed, and must therefore have diminished the difference according to the extent to which the cause operated. These observations therefore show beyond doubt, that the length of a needle has a very sensible influence on the extent of its deviations. Although I did not consider that the difference in the masses of the needles had much effect in diminishing the difference of their deviations in some cases, or in increasing it in others, yet in order to decide what part of the difference was due to mass, and what to length, it was my intention, could I have found leisure, to have made observations corresponding to the foregoing with the needles A and P, and

two others six inches in length, one extremely light and the other much heavier than A.

In order to compare the theoretical results which we have obtained, that is the equations (3) and (6), with the observed deviations of the three needles, I take a mean of the deviations with the shell below the horizontal plane of the needle, and with it above, when the azimuth of its centre from south in the second case was the same as its azimuth from north in the first, as the values of φ corresponding to the different azimuths with the shell's centre below the horizontal plane of the needle: and likewise with the shell's centre in the horizontal plane of the needle, I take a mean of the deviations in the corresponding azimuths from north and south as the values of φ in this case. From the values of h, r, η*, θ are derived those of γ and ω, and thence, from the values of φ, may be computed the corresponding values of ψ or the deviations of a freely suspended needle; and the value of the constant $\frac{2\,m\,R^3}{fe}$ will be determined from the equation,

$$\text{Tan}\,\psi = \frac{3 \sin 2\gamma}{\frac{2\,m\,R^3}{fe} + 3 \cos 2\gamma + 1};$$

or we have

$$\frac{2\,m\,R^3}{fe} = \frac{6\,r}{R^2}(r \cos\theta + h \cot\eta) \, . \, \frac{\sin(\theta - \varphi)}{\sin\varphi} + 2,$$

from the equation (6), without the previous computation of γ, ω, ψ. The values of γ and ω corresponding to the different adjustments of the shell, the values of ψ and of the constant $\frac{2\,m\,R^3}{fe}$, computed from the means of the observed values of φ, with the different needles, are contained in the following Table.

The vertical distance of the centre of the Shell from that of the needle or $h = 10$ inches, $r = 13.5$ inches, and $R = 16.8$ inches.

	$\theta =$	20° 00′ 00″	40° 00′ 00″	60° 00′ 00″	80° 00′ 00″	100° 00′ 00″	120° 00′ 00″	140° 00′ 00″	160° 00′ 00″	
	$\gamma =$	35 09 21	39 39 30	45 50 00	52 37 23	59 13 45	65 02 40	69 35 11	72 28 41	
	$\omega =$	28 30 34	54 01 48	75 57 51	95 13 54	112 55 33	129 51 52	146 33 20	163 14 58	Mean values of $\frac{2\,m\,R^3}{fe}$
Deviations with	A =	10 33 50	20 14 35	28 01 35	32 32 05	32 12 45	27 55 05	21 43 10	13 06 10	
	D =	9 46 35	18 51 05	26 28 55	31 36 30	32 55 50	29 29 45	21 26 35	10 51 55	
	P =	9 44 15	18 46 50	26 22 40	31 32 40	32 58 45	29 32 25	21 27 00	10 43 40	
Values of ψ with	A =	11 09 37	11 45 42	12 06 19	11 43 59	10 35 54	9 28 04	8 57 59	9 05 19	
	D =	9 58 36	10 39 07	11 15 00	11 21 01	10 49 44	9 54 08	8 53 14	8 06 22	
	P =	9 55 13	10 35 49	11 11 37	11 19 27	10 50 40	9 54 52	8 53 21	8 02 27	
Values of $\frac{2\,m\,R^3}{fe}$ with	A =	12.307	12.602	13.069	13.725	14.525	14.694	13.701	12.225	13.356
	D =	14.047	14.118	14.163	14.208	14.218	14.079	13.813	13.551	14.025
	P =	14.140	14.201	14.241	14.242	14.198	14.063	13.810	13.650	14.068

* I take $\eta = 20°$, the most recent observation giving the dip at this place 70° 00′.

The centre of the Shell in the same horizontal plane as that of the Needle or $h = 0$, and $R = 16.68$ inches.

	$\theta =$	20° 00′ 00″	40° 00′ 00″	60° 00′ 00″	80° 00′ 00″	Mean values of $\frac{2mR^3}{fe}$
	$\gamma =$	71 15 10	74 48 40	80 09 12	86 35 42	
	$\omega =$	21 10 22	41 45 48	61 31 08	80 35 31	
Deviations with	A =	7 22 10	11 55 55	10 43 45	4 05 30	
	D =	6 16 35	10 28 50	10 21 10	4 29 15	
	P =	6 12 25	10 24 25	10 18 55	4 31 05	
Values of ψ with	A =	10 07 10	7 56 38	4 39 57	1 26 12	
	D =	8 05 06	6 43 38	4 28 55	1 34 40	
	P =	7 57 51	6 40 04	4 27 49	1 35 19	
Values of $\frac{2mR^3}{fe}$ with	A =	11.612	12.460	14.212	16.163	13.612
	D =	14.235	14.450	14.722	14.893	14.575
	P =	14.433	14.565	14.775	14.804	14.644

From these values of $\frac{2mR^3}{fe}$, with the different needles, it is evident that if the mean value with the needle A were substituted in the equation (6), the computed values of ϕ would not differ greatly from the observed values; but that with the needles D and P, the approximation would be so close, that the differences would clearly be within the limits of errors of observation. There can therefore be no doubt that the law which I proposed will give an extremely close approximation to experimental results, when the length of the needle does not bear too sensible a proportion to its distance from the centre of the attracting mass of iron; and that even if this ratio is not less than 1 to 3, the theoretical and experimental results will not differ greatly from each other.

I will now, for the purpose of comparison, deduce from these experiments the values of the constant depending upon the law, that "the tangent of the deviation is proportional to the rectangle of the cosine of the longitude, and the sine of the double latitude." If λ is the latitude of the needle's centre from a great circle of the shell perpendicular to the dip; and l its longitude measured from the intersection of this circle with the horizontal plane through the shell's centre; δ the dip of the needle; and M a constant,—then the law is

$$\text{Tan } \phi = \frac{\sin 2\lambda \cos l}{M \cos \delta}:$$

in which equation λ, l, δ are the complements of γ, ω, η in the foregoing equations.

The values of M determined by means of this equation from the mean of the

observed values of ϕ in the different azimuths, with the three needles, are contained in the Tables below.

The vertical distance of the centre of the Shell from that of the Needle or $h = 10$ inches, $r = 13.5$ inches, and $R = 16.8$ inches.								
$\theta=$	20°	40°	60°	80°	100°	120°	140°	160°
Values of M with A=	21.137	18.916	15.980	13.211	11.273	9.721	7.934	6.236
Values of M with D=	22.877	20.431	17.074	13.694	10.966	9.105	8.047	7.566
Values of M with P=	22.970	20.514	17.152	13.728	10.945	9.089	8.044	7.662

The centre of the Shell in the same horizontal plane as that of the Needle or $h = 0$, and $R = 16.8$ inches.				
$\theta=$	20°	40°	60°	80°
Values of M with A=	14.910	13.981	13.712	14.344
Values of M with D=	17.533	15.971	14.222	13.074
Values of M with P=	17.731	16.086	14.275	12.986

Here it appears that the values of M derived from these experiments, instead of being constant or nearly so, as they ought to be, vary from 6 to 21 with the long needle, and with the short ones from 8 to 23. The results are most consistent when the centres of the shell and needle are in the same horizontal plane; and within certain limits of the values of l, l being likewise constant throughout a set of observations, results that should not differ so very widely as the foregoing might be obtained: it is somewhat singular that the truth of the law should have been considered to be established experimentally from observations made in precisely these relative positions of the shell and needle*. When the centre of the shell and needle are in the same horizontal plane, the values of M deduced from the observations with the long needle A, agree much more nearly with each other than those deduced from the observations with either of the shorter needles D or P; and when they are not in the same plane, the disagreement in the values of M is nearly the same with each of the needles: we may therefore infer, that the proximity of the needle to the shell is not the cause of this disagreement, and that the same would be found to be the case if the values of M were deduced from deviations of the needle observed at greater distances from the shell's centre.

* Mr. Barlow's Magnetic Attractions, 2nd ed. pp. 34, 37, 38.

We must therefore conclude, that although the law that "the tangent of the deviation is proportional to the rectangle of the cosine of the longitude and the sine of the double latitude" has been derived from experiment, and considered as giving close approximations to experimental results*; yet as it is quite clear that no law giving such inconsistent results as appear here† can be considered as even approximately true, it must be wholly rejected.

On the principle which I have assumed, I have already given the general explanation of the phænomena observed with needles having their magnetism unequally distributed; I shall now describe the experiments which I made with such needles, with the view of comparing them with the equations that I should derive from this principle, according to the distribution of magnetism which I found to take place under different circumstances of disturbance, which equations, (10) and (11), have been already given. In the beginning of this paper I have described the needles made use of in these experiments, and the manner of magnetizing them. These needles are distinguished by the letters A, B, C; A being that made use of in the experiments already described, and B and C of precisely the same form and weight as A.

In order to determine nearly the situations of the magnetic centre, and of the points towards which the forces are directed near the ends, in these needles, which points may be considered as their poles nearly, I successively placed them on a wooden rectangular scale, so that their axes coincided with a line drawn through the middle parallel to its sides. This scale, which is graduated

* Mr. Barlow's Magnetic Attractions, 2nd ed. pp. 38, 39.

† If we take a mean of the deviations with the undeteriorated needle in the observations given in the Transactions of last year, p. 285, in the corresponding azimuths with the shell above and below the table, the values of M will be these:

$\theta =$	20°	40°	60°	80°	100°	120°	140°	160°
M =		17.65	14.51	12.38	10.81	10.22	8.51	6.61

The observation corresponding to $\theta = 20°$, which would have given by much the greatest value of M, does not appear to have been made. These values are not quite so discordant as those resulting from my observations, but are equally conclusive against the law from which they are derived. The means of the deviations when the shell was in the same horizontal plane as the needle, give the following values of M.

$\theta =$	20°	40°	60°	80°
M =	14.40	13.33	12.91	12.83,

which are rather more discordant than the corresponding values from my observations.

across to tenths of an inch, and can be made to revolve about a vertical axis, being placed horizontally so that the axis of the needle was at right angles to the meridian, a rectangular stand, carrying the small needle P level with the other, was passed along and in contact with the northern side of the scale, until the axis of P pointed accurately in the meridian. This occurred in two positions; in the one, the force of the needle under trial concurring with the terrestrial directive force; and in the other, being in direct opposition to it. The distances, from the centre of the needle, of the points to which the centre of P was in these cases opposite were noted; and the same was done when P was passed to the south of the rectangular scale: the mean of the respective distances gave the distances, from the centre, of the points towards which the forces are directed, or nearly those of the poles. The position of the magnetic centre was determined similarly, by placing the rectangular scale, carrying the needle under trial, so that the axis of this needle was in the magnetic meridian with the marked end towards north, and passing P along the eastern side until its axis was in the meridian with its marked end south, and then along the western side. In all cases the distance between the axis of the needle and the centre of P was 1.5 inch.

The positions of these points in each needle, so ascertained after it had been carefully magnetized by double touch, as I have before described, and the times of the needle's vibration, were as follow.

Needle.	Distances from the Needle's centre.			Time of making 10 vibrations.
	Of the south or marked pole.	Of the magnetic centre or zero.	Of the north or unmarked pole.	
	inches.	inches.	inches.	sec.
A.	2.43	0.00	2.42	34.1
B.	2.39	0.04 U.	2.42	32.75
C.	2.35	0.07 U.	2.47	32.7

After making these observations, the magnetism of the needle A was undisturbed; that of B was disturbed by drawing the unmarked end of a 12-inch bar magnet from the centre to the unmarked end of the needle twice, a thin board being interposed; that is, the needle B had its "southern branch deteriorated;" and the needle C had its "northern branch deteriorated," by similarly drawing the marked end of the magnet twice from the centre to the marked

end. The situations of the poles and magnetic centre were ascertained, as before, immediately after making the first set of observations, shortly to be described, on the deviations of these needles, when the shell was above the table on which they were placed; likewise previously to commencing the second set, when the shell was below the table; and also after the observations with the shell's centre in the plane of the table were concluded. The time in which each needle made ten vibrations was likewise observed. The following are the results.

Needle.	Distances from the Needle's centre.			Time of making 10 vibrations.	Date.	Hour.
	Of the south or marked pole.	Of the magnetic centre or zero.	Of the north or unmarked pole.			
	inches.	inches.	inches.	sec.		
C.	1.41	1.06 U.	2.80	51.8	28 Nov.	6 P.M.
	1.43	1.06	2.80	50.1	3 Dec.	1 P.M.
	1.49	1.04	2.80	50.65	5 Dec.	3 P.M.
Means	1.44	1.05	2.80	50.85		
A.	2.40	0.05 U.	2.44	35.1	28 Nov.	6 P.M.
	2.41	0.03	2.43	34.35	3 Dec.	1 P.M.
	2.42	0.025	2.43	35.3	5 Dec.	8 P.M.
Means	2.41	0.035	2.433	34.92		
B.	2.74	0.71 M.	2.23	52.85	28 Nov.	6 P.M.
	2.73	0.70	2.26	51.4	3 Dec.	1 P.M.
	2.72	0.62	2.33	51.8	5 Dec.	11 A.M.
Means	2.73	0.677	2.273	52.02		

In these the letter M placed after the distance of zero from the needle's centre, indicates that the distance was measured towards the marked end of the needle; and the letter U, that it was measured towards the unmarked end. The distances of the poles and magnetic centres, as determined on different days, show that only a small change took place in the situations of these points in the needles during the time of making the observations on their deviations*.

To determine the intensity of the free magnetism on each side of the centres

* With the view of ascertaining what degree of permanency this disturbed state of the magnetism in a needle may have, I made observations similar to these, at considerable intervals of time, with the two 9-inch bar magnets which I have already mentioned. The bars had been carefully and strongly magnetized by double touch on the 5th of November, when the positions of the poles and magnetic centres appeared to be as in the first observations to each magnet, on that day, in the following Table. The magnetism of the bars was then disturbed by passing the unmarked end of a 12-inch magnet from the centre of I, to its unmarked end, and the marked end of the same magnet from the centre to the

of force, and likewise the position of the points in these needles where the intensities of contrary names were equal, I adopted the method of COULOMB. For this purpose a small needle 0.72 inch in length, 0.15 inch in breadth, of a form somewhat similar to A, B, or C, and weighing 1.25 grain, which for distinction I name I, was suspended by a single fibre of the silk-worm within a glass cylinder attached to a high rectangular support of wood, in which is

marked end of II: after this, the second observations on the 5th of November were made. In the interval between these observations and the subsequent ones, care was taken that these bars were quite out of the reach of accidental disturbance in their magnetism.

Magnet.		Distances from the Bar's centre. Of the south or marked pole.	Of the magnetic centre or zero.	Of the north or unmarked pole.	Date of observation.
		inches.	inches.	inches.	
I.	Magnetism undisturbed	3.75	0.48 U.	3.80	5 Nov. 1827
	Magnetism disturbed at unmarked end......	4.06	1.49 M.	2.73	5 Nov.
		4.10	1.31	2.84	12 Dec.
		4.12	1.32	2.86	23 April, 1828
		4.12	1.32	2.86	14 May
II.	Magnetism undisturbed	3.73	0.01	3.76	5 Nov. 1827
	Magnetism disturbed at marked end......	2.64	1.52	4.05	5 Nov.
		2.67	1.47	4.07	12 Dec.
		2.75	1.45	4.05	23 April, 1828
		2.82	1.37	4.05	14 May

It appears that some, though an inconsiderable change took place in the distribution of the magnetism of the bar I, from the 5th of November to the 12th of December, but that it has remained almost precisely in the same state from the 12th of December to the 14th of May; and it will probably continue in this state so long as it meets with no extraneous disturbance. In the latter interval the observations indicate some change in the magnetism at the marked end of II; but even here it is not to a great extent. In neither of these bars are there any indications of other poles than those whose situations I observed. I am not aware of any experiments having been made to ascertain whether in magnetized steel bars the action of the magnetic particles upon each other has a tendency to restore a symmetrical arrangement of their magnetism when such arrangement has been disturbed; but the results I have obtained show that if such tendency exist for a short time after this disturbance, it soon, at least in some cases, either ceases to exist, or is so feeble as not to overcome, or to overcome very slowly, the coercive force of the steel. In the bars which I employed, this coercive force could not be great, if it depend upon hardness, since they are softer than requisite for working with a file. We ought, however, to be in possession of more extended observations before we draw any general conclusions from such facts as I have stated.

an opening from top to bottom parallel to its sides, of the same size as the stems of two clamping screws fixed to the rectangular scale, before mentioned, to which the needle under trial was attached. This rectangular support was rendered vertical by means of foot-screws, and a level attached to it; so that when the scale was applied to it, the axis of the needle to be examined was likewise vertical; and the instrument was adjusted so that the axis of this needle and the centre of the needle I, were in the magnetic meridian. In the same horizontal plane as the centre of I, was an index, which marked on the scale the point of the needle on it horizontally opposite to that centre. As the stems of the clamping screws fitted the opening in the support, the scale and the axis of the large needle could be moved vertically, so that any required point of this needle should be opposite to the centre of I: when the scale had been so adjusted, it was clamped to the support. The needle I was then vibrated 45° from 0, and the time of making 100 vibrations being carefully noted, the force accelerating I was known. The terrestrial force accelerating I having been determined previously to applying the larger needle, the intensity of the force by which this needle accelerated I, and consequently nearly the inten-

	From the Needle's centre of figure towards its marked end.											
Needle C, having had its magnetism disturbed in the marked or northern branch.	Distance of the point opposite to the needle I	2.99 in.	2.80 in.	2.60 in.	2.40 in.	2.20 in.	2.00 in.	1.80 in.	1.60 in.	1.40 in.	1.20 in.	1.00 in.
	Time in which the needle I made 100 vib^{ns}.	70.4^s	67.8^s	65.5^s	63.8^s	61.95^s	60.3^s	59.7^s	58.9^s	58.1^s	57.9^s	57.7^s
	Intensity of force accelerating I	+2.9933	+3.2273	+3.4579	+3.6447	+3.8656	+4.0800	+4.1624	+4.2763	+4.3948	+4.4253	+4.4560
	Intensity of magnetism in C.....................	+1.9933	+2.2273	+2.4579	+2.6447	+2.8656	+3.0800	+3.1624	+3.2763	+3.3948	+3.4253	+3.4560
Needle A, having its magnetism similarly distributed in the two branches.	Distance of the point opposite to the needle I	2.99 in.	2.80 in.	2.60 in.	2.42 in.	2.20 in.	2.00 in.	1.80 in.				
	Time in which the needle I made 100 vib^{ns}.	43.25^s	40.8^s	39.8^s	39.25^s	39.25^s	39.45^s	40.1^s				
	Intensity of force accelerating I	+7.8785	+8.8535	+9.3040	+9.5666	+9.5666	+9.4699	+9.1653				
	Intensity of magnetism in A.....................	+6.8785	+7.8535	+8.3040	+8.5666	+8.5666	+8.4699	+8.1653				
Needle B, having had its magnetism disturbed in the unmarked or southern branch.	Distance of the point opposite to the needle I	2.99 in.	2.80 in.	2.72 in.	2.60 in.	2.50 in.	2.40 in.				0.70 in.	0.625 in.
	Time in which the needle I made 100 vib^{ns}.	55.3^s	53.8^s	53.4^s	53.2^s	53.2^s	53.5^s				112.2^s	121.4^s
	Intensity of force accelerating I..............	+4.8590	+5.1338	+5.2110	+5.2503	+5.2503	+5.1916				+1.1804	+1.0083
	Intensity of magnetism in B....................	+3.8590	+4.1338	+4.2110	+4.2503	+4.2503	+4.1916				+0.1804	+0.0083

sity of the magnetism in its different points, became known. The needle I was rendered extremely hard, and was strongly magnetized, in order that its magnetism should not be disturbed by that of the large needle. The distance between the centre of I and the axis of the large needle was in all cases 1.63 inch, so that in some positions the vibrations of I, with its own inertia alone to overcome, would have been too rapid for observation; and to obviate this, it had attached to it, in all the observations, a thin disc of mica. The needle I was vibrated to the north of the large needle, whose marked or south pole was always downwards. The time of vibration of I, corresponding to each point of the large needle was determined by two trials, and as these never differed by more than a fifth of a second, the time may be considered to have been determined at least within this limit. The following Table shows the times of vibration of I, when not affected by the needles A, B, C, and when points at different distances from the centres of these needles were horizontally opposite to the centre of I; also the intensity of the magnetism in these points, deduced from those times of vibration; + indicating south polarity, or that predominating in the marked end of the needle, and — indicating north polarity.

			From the Needle's centre of figure towards its unmarked end.										Vibrating by the terrestrial force alone.
0.80 in.	0.60 in.	0.00 in.	1.035 in.	1.10 in.	1.12 in.		2.00 in.	2.20 in.	2.40 in.	2.60 in.	2.80 in.	2.99 in.	
57.7^s	58.75^s		107.7^s	120.4^s	126.0^s		73.1^s	65.9^s	61.8^s	59.7^s	59.8^s	61.6^s	121.8^s
+4.4560	+4.2981		+1.2790	+1.0234	+0.9345		−2.7763	−3.4160	−3.8843	−4.1624	−4.1485	−3.9096	1.0000
+3.4560	+3.2981		+0.2790	+0.0234	−0.0655		−3.7763	−4.4160	−4.8843	−5.1624	−5.1485	−4.9096	
		0.00 in.	0.025 in.	0.05 in.	0.075 in.	1.80 in.	2.00 in.	2.20 in.	2.43 in.	2.60 in.	2.80 in.	2.99 in.	
		108.6^s	115.6^s	120.1^s	129.2^s	47.3^s	46.2^s	45.9^s	45.6^s	45.8^s	48.1^s	50.2^s	121.4^s
		+1.2496	+1.1029	+1.0218	+0.8829	−6.5874	−6.9048	−6.9954	−7.0877	−7.0260	−6.3701	−5.8483	1.0000
		+0.2496	+0.1029	+0.0218	−0.1171	−7.5874	−7.9048	−7.9954	−8.0877	−8.0260	−7.3701	−6.8483	
0.615 in.	0.60 in.				2.00 in.	2.10 in.	2.20 in.	2.33 in.	2.40 in.	2.60 in.	2.80 in.	2.99 in.	
122.8^s	126.8^s				84.2^s	83.55^s	83.25^s	83.50^s	83.85^s	85.5^s	89.0^s	94.1^s	121.9^s
+0.9854	+0.9242				−2.0960	−2.1287	−2.1441	−2.1313	−2.1135	−2.0327	−1.8760	−1.6781	1.0000
−0.0146	−0.0758				−3.0960	−3.1287	−3.1441	−3.1313	−3.1135	−3.0327	−2.8760	−2.6781	

From these results it appears, that the points previously determined as the centres of force are not, in the needles C and B, precisely those in which the intensities are the greatest, but that the situations of the magnetic centres determined by the two methods very nearly agree; the distance of this centre from that of figure being, for the needle C, 1.105 inch towards the unmarked end, and for the needle B, 0.6214 towards its marked end. As might have been anticipated, the intensities of the poles were changed, as well as their situations, by the disturbance of the magnetism in either branch; and this not only in the branch to which the disturbing magnet had been applied, but also in the other. In the branch to which the magnet had been applied, the intensity was reduced in about the ratio of 83 to 31 in the needle B, and of 83 to 35 in the needle C; and in the other branch, in the ratio of 83 to 43 in B, and of 83 to 52 in C. In both cases the least intense pole was much more diffused than the other; the intensity in the branch to which the magnet had been applied changing but little over a space of nearly two inches. It appears likewise, that at the points previously determined as the centres of force in the two branches of each needle, the intensity multiplied by the distance from the magnetic centre is nearly the same on each side of that centre; these products on the contrary sides being 8.9 and 9.2 with the needle B, and with C, 8.7 and 8.8.

The observations on the deviations of the three needles, A, B, C, with their magnetism thus distributed, produced by the 18-inch shell, are contained in the following Table: they were made in precisely the same manner as those already described, with the needles A, D, P; the deviation of one needle having been observed for any azimuth, it was removed from the pivot and replaced by another, and this again by the third without moving the pivot after it had been adjusted to the required position: the vertical and horizontal distances of the shell were likewise the same as in the former case.

Needle of which the deviation was observed.	Azimuth of the Shell's centre from North.	The centre of the Shell 16.8 inches from the Needle's centre, and								
		10 inches below the horizontal plane passing through that centre.			In the horizontal plane passing through that centre.			10 inches above the horizontal plane passing through that centre.		
		Azimuth of the Shell's centre. West. Deviations.	Azimuth of the Shell's centre. East. Deviations.	Means of the deviations with the Shell West and with the Shell East.	Azimuth of the Shell's centre. West. Deviations.	Azimuth of the Shell's centre. East. Deviations.	Means of the deviations with the Shell West and with the Shell East.	Azimuth of the Shell's centre. West. Deviations.	Azimuth of the Shell's centre. East. Deviations.	Means of the deviations with the Shell West and with the Shell East.
C, having had its magnetism disturbed in the marked or northern branch.	20°	9° 13′W	8° 57′E	+ 9° 05′ 00″	5° 38′W	5° 20′E	+ 5° 29′ 00″	7° 06′E	6° 42′W	− 6° 54′ 00″
	40	17 07	17 02	17 04 30	9 03	8 54	8 58 30	14 33	13 57	14 15 00
	60	23 57	23 37	23 47 00	9 14	8 33	8 53 30	24 26	24 05	24 15 30
	80	27 48	28 04	27 56 00	4 58	4 27	4 42 30	35 36	35 20	35 28 00
	100	29 34	29 50	29 42 00	2 41E	3 05W	− 2 53 00	38 28	38 33	38 30 30
	120	29 30	29 26	29 28 00	12 22	12 47	12 34 30	32 56	33 07	33 01 30
	140	27 27	27 08	27 17 30	14 45	15 15	15 00 00	23 17	23 51	23 34 00
	160	21 21	21 19	21 20 00	8 54	9 22	9 08 00	11 46	12 23	12 04 30
A, having its magnetism similarly distributed in the two branches.	20	10 50	10 36	10 43 00	7 18W	7 05E	+ 7 11 30	12 53	12 22	12 37 30
	40	20 30	20 18	20 24 00	11 38	11 36	11 37 00	21 18	20 46	21 02 00
	60	28 24	28 09	28 16 30	10 30	10 04	10 17 00	27 39	27 30	27 34 30
	80	32 22	32 47	32 34 30	3 43	3 28	3 35 30	32 12	31 57	32 04 30
	100	31 54	32 32	32 13 00	4 28E	4 44W	− 4 26 00	32 28	32 23	32 30 30
	120	27 54	28 10	28 02 00	11 20	11 35	11 27 30	28 09	28 02	28 05 30
	140	21 48	21 50	21 49 00	12 20	12 38	12 29 00	20 10	20 31	20 20 30
	160	13 18	13 10	13 14 00	7 30	7 48	7 39 00	10 18	10 41	10 29 30
B, having had its magnetism disturbed in the unmarked or southern branch.	20	11 40	11 32	11 36 00	7 53W	7 50E	+ 7 51 30	17 58	17 12	17 35 00
	40	22 06	22 03	22 04 30	12 30	12 35	12 32 30	24 53	24 24	24 38 30
	60	30 50	30 37	30 43 30	10 33	10 12	10 22 30	28 37	28 25	28 31 00
	80	34 52	35 32	35 12 00	2 42	2 31	2 36 30	30 30	30 12	30 21 00
	100	33 06	33 56	33 31 00	4 58E	5 04W	− 5 01 00	29 45	29 23	29 34 00
	120	26 24	26 52	26 38 00	10 20	10 23	10 21 30	25 24	25 15	25 19 30
	140	19 00	19 08	19 04 00	11 00	11 10	11 05 00	18 26	18 34	18 30 00
	160	10 40	10 36	10 38 00	6 46	6 54	6 50 00	9 22	9 41	9 31 30

The deviations of A in this Table, are the values of ϕ in the equations (10) and (11): the deviations of C and B are the corresponding values of $\phi_{,}$ the value of k being plus in the first case, and minus in the second. To compare these equations with the observations, it is necessary that the value of p should be known as well as that of k. This may be determined from the equations by means of the observed values of $\phi_{,}$ and ϕ in the above table; but the computations from the equation (10) would be so laborious, that I must acknowledge I have not, for this reason, entered upon them. I have however computed the values of p from the equation (12), when the centre of the shell was in the same horizontal plane as that of the needle, although these computa-

tions are sufficiently tedious: and from the mean of the values thus determined, I have likewise computed the values of φ, or the deviations of A, both from the observations with C and from those with B, by means of the equation (11), taking $k = 1.105$ in the one case, and $k = -0.6214$ in the other, as determined from the times of vibration of the Needle I.

The following TABLE contains the results thus obtained.

The centre of the Shell in the same horizontal plane as that of the Needle.										
	The deviations of A deduced from those of C.					The deviations of A deduced from those of B.				
Azimuth of the Shell's centre from north, or values of θ.	Observed values of $\varphi_,$.	Observed values of φ.	Values of p, computed from the equation (12), making $k=1.105$.	Values of φ, computed from the equation (11), making $p=1.0782$.	Difference between the observed and computed values of φ.	Observed values of $\varphi_,$.	Observed values of φ.	Values of p, computed from the equation (12), making $k=-0.6214$.	Values of φ, computed from the equation (11), making $p=1.0551$.	Difference between the observed and computed values of φ.
20°	5° 29′	7° 11½′	1.1256	6° 57′	−14½′	7° 51½′	7° 11½′	1.0289	7° 20′	+08½′
40	8 58½	11 37	1.1180	11 15	−22	12 32½	11 37	1.0314	11 51	+14
60	8 53½	10 17	1.0654	10 25	+08	10 22½	10 17	1.0433	10 24	+07
80	4 42½	3 35½	1.0617	3 39	+03½	2 36½	3 35½	1.1617	3 14	−21½
100	2 53	4 36	1.2045	4 03	−33	5 01	4 36	1.0781	4 29	−07
120	12 34½	11 27½	1.0116	12 13	+45½	10 21½	11 27½	1.0489	11 30	+02½
140	15 00	12 29	1.0095	13 12	+43	11 05	12 29	1.0306	12 44	+15
160	9 08	7 39	1.0294	7 55	+16	6 50	7 39	1.0176	7 52	+13
Mean value of p			1.0782			Mean value of p		1.0551		

The deviations of A computed from those of B, agree with the observed deviations of A quite as nearly as we can expect; but those computed from the deviations of C do not approximate so closely to the observations. The difference in the values of φ and $\varphi_,$ principally arises from the value of k, and any error in determining this will have a corresponding effect in the values of φ determined from those of $\varphi_,$. From the results here, the value of k for the needle B appears to have been determined with considerable accuracy; and I am not aware of any circumstances in the observations that should have rendered it less so for the needle C. It is however to be observed, that the

magnetic centre will not be precisely the same for all distances at which the needle may act, and that the position of this point will vary the more, the greater is its distance from the centre of figure of the needle. If for the needle C we increase the value of k by one-tenth, that is making it 1.2155 instead of 1.105, all but one of the computed values of ϕ will agree more closely with the observed values, and this one will still be extremely near to the corresponding observed value. This will be seen by the following Table computed as the preceding, excepting that, in taking the mean of the values of p, I omit that corresponding to $\theta = 100°$, and which I have done because a very small error of adjustment, whether arising from the compass not being placed exactly in the required position, or from the centre of the shell not occupying accurately the centre of the circle to which the compass was adjusted, would here have a most sensible effect upon the deviations; and such appears to have been the case, from the difference in the observed values of ϕ, when θ was 80°, and when θ was 100°.

The centre of the Shell in the same horizontal plane as that of the Needle.					
	The deviations of A deduced from those of C.				
Azimuth of the Shell's centre from north, or values of θ.	Observed values of $\phi_{\prime}$.	Observed values of ϕ.	Values of p, computed from the equation (12), making $k = 1.2155$.	Values of ϕ, computed from the equation (11), making $p = 1.069$.	Difference between the observed and computed values of ϕ.
20°	5° 29′	7° 11½′	1.1037	7° 01′	−10½′
40	8 58½	11 37	1.0996	11 20	−17
60	8 53½	10 17	1.0570	10 24	+07
80	4 42½	3 35½	1.1031	3 28	−07½
100	2 53	4 36	(1.1807)	4 05	−32
120	12 34½	11 27½	1.0224	11 59	+32
140	15 00	12 29	1.0354	12 49	+20
160	9 08	7 39	1.0617	7 41	+02
		Mean value of p	1.0690		

Considering that the equation (6), from which (11) is derived, does not

afford a close approximation at this distance of the shell when the length of the needle is six inches, as appears from the observations with the needles A, D, P; and that in deducing this equation, the approximation was limited to the first power of $\frac{k}{R}$, I scarcely anticipated so close an agreement between the observed deviations of the undeteriorated needle A, and the deviations of such a needle computed from the observed deviations of the deteriorated needles B and C: this agreement has therefore fully confirmed the views which I originally took of this subject, as stated in a note to my paper in the Transactions of last year. There can be no doubt that the equation (11) affords the proper correction by which to obtain the values of φ from those of $\varphi_{,}$, when the centre of the shell is in the same horizontal plane as that of the needle; and the equation (10) will give the same correction when its centre is not in that plane. I have given the observations requisite for comparison; but this comparison I must leave to others who may be interested in the inquiry, and have more leisure for these tedious computations than the duties of my situation have allowed me.

Instead then of these "secondary deflections," as they have been termed, due to the deterioration of the needles, being inconsistent with the hypothesis "which attributes the deflection of a magnetized needle to the general central attraction of the iron on an imaginary needle passing through the pivot (centre) in the line of the dip," as is assumed in the paper giving an account of experiments with deteriorated needles, I have shown I think clearly, not only that they are consistent with it, but that this hypothesis affords the proper corrections to observations with such imperfect instruments; and therefore additional weight is given to the arguments which might have been previously adduced in support of this hypothesis. Should there be any still disposed to think that the hypothesis of induced magnetism will give closer approximations to the observed deviations than I have derived from that of a central action of the iron, the determinations which I have given of the distribution of the magnetism in these needles, will enable them to compute the deviations on that hypothesis, and to exhibit the resulting approximations.

I have, however, no intention of controverting the hypothesis of induced magnetism. That hypothesis undoubtedly affords satisfactory explanations of

many magnetical phænomena; but there is one experiment, and that a very decisive one, which appears quite inconsistent with it; and until that experiment can be clearly explained on the hypothesis, that iron acts by induced magnetism alone, I think some reservation is necessary in our assent to that hypothesis. According to it, the lower part of a bar of steel or iron is a south pole, and the upper a north pole; and when the ends of the bar are reversed, the poles are immediately so likewise. We might admit such change of the poles in the case of soft steel or iron, but by no means in that of hard steel. The following experiment of Mr. Barlow's requires the same admission in both cases: "I procured a steel bar three feet long, an inch broad, and half an inch thick, and had it rendered, according to the expression of the workmen, "as hard as fire and water could make it;" and I must say that I was not at all surprised to find that it produced precisely the same effect as the softest iron, changing its poles with its position (to adopt the language of our author, Biot,) with equal facility*;" that is, I conceive, the deviations of a needle corresponding to such change. As the rapid change in the poles, which is here necessary to suppose, on the hypothesis of induction from the earth, is quite at variance with the phænomena observed on the approach of a magnet to a bar of hard steel, and its subsequent removal, it is necessary that the cause of this difference should be clearly shown, before we admit that iron or steel has no action upon a needle, but that which arises from the separation of its magnetism by the influence of the earth or of the needle. When a mass of iron is so far removed from a magnetized needle that the magnetism induced in it from the needle produces but little effect, I have shown that the deviations computed on the hypothesis of a central attraction, agree with those observed, even in cases which were supposed directly opposed to such an hypothesis; and therefore it is not necessary to suppose that the iron is polarized by the action of the earth.

However, whether we admit the hypothesis of a central attraction independent of the action by induction when the iron is in the immediate neighbourhood of a magnet, or not, this hypothesis has the advantage of very readily connecting all the phænomena observed in the deviations of a mag-

* Magnetic Attractions, p. 124, 1st edition: the experiment is omitted in the 2nd edition.

netized needle which are due to the action of iron; of indicating immediately the nature of the deviations, even under very complicated circumstances, as I have shown in the case of the deteriorated needles; and still further of affording ready and close approximations to numerical results in all positions within certain limits of distance from the attracting body.

S. H. CHRISTIE.

Royal Military Academy,
22nd May, 1828.